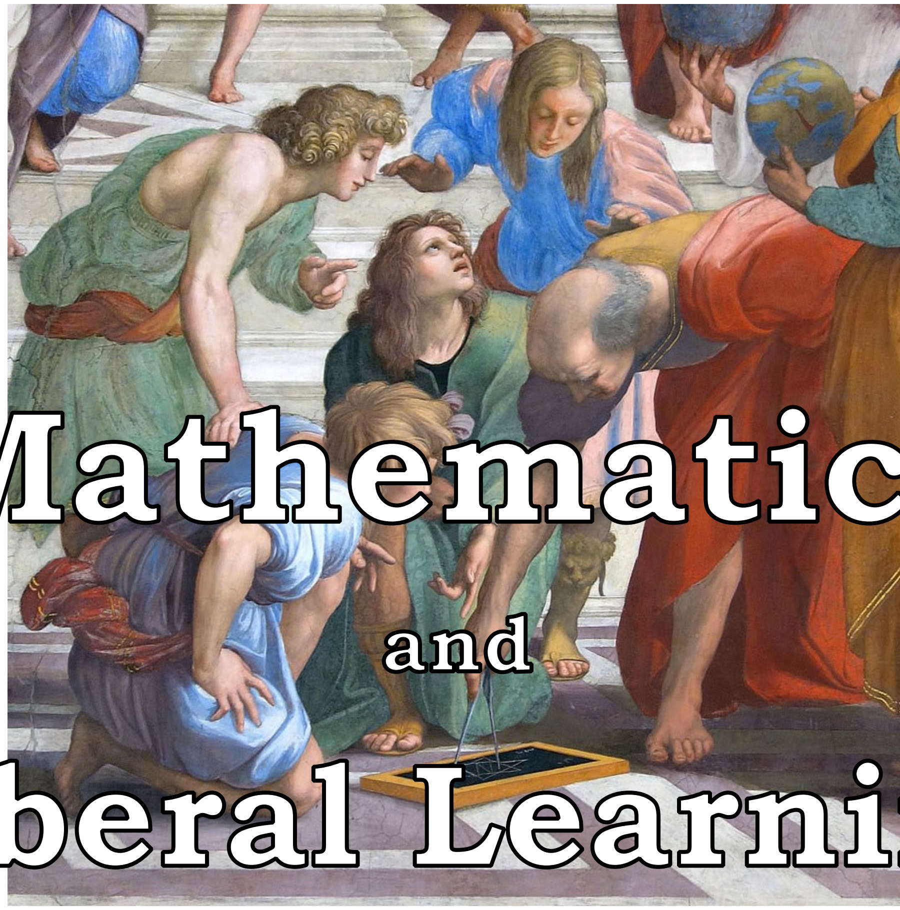

Mathematics and Liberal Learning

Opening Lecture

for the Second Academic Year of

Thomas Aquinas College

New England

September 4, 2020
Dolben Auditorium, 7:30 pm

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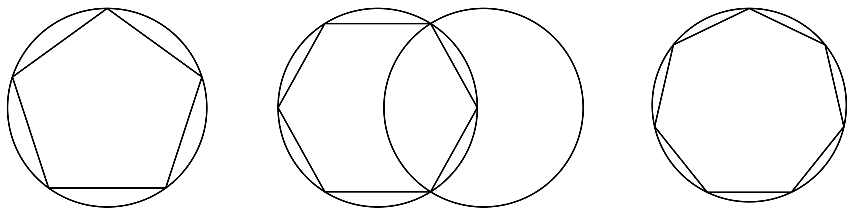

Introduction

Our college offers no education other than a liberal one. Some who are aware of that fact are surprised, even horrified, to learn that we include four years of mathematics in our program. Why, they wonder, should liberal education include mathematics at all, let alone four years of it? Isn't liberal education restricted to those disciplines known as the humanities, which by definition (and by the goodness of God) do not include mathematics and science? Shouldn't liberal education confine itself to history, literature, philosophy, and the like?

The truth, however, is that mathematics is an integral part of liberal education. By an *integral part* I mean not merely a component, such as a brick in a brick wall, but a part that, like an organ of an animal, is specially adapted to all the other components so as to serve them or be served by them for the good of the whole.

Mathematics belongs in liberal education first of all because it is itself a liberal discipline, and so it is indeed a component of liberal education; but more than that, it also serves all superior components of liberal learning, namely, natural science, philosophy, and sacred theology.

There you have my enunciation; I am now obliged to supply a proof of it. Since my enunciation has two parts, my talk will fall into two corresponding parts. The first will show deductively that mathematics is itself a liberal discipline. The second will show inductively that mathematics in various ways serves the liberal disciplines superior to itself, and thus serves liberal education as a whole.

Part 1: Mathematics as A Liberal Art and Science

On, then, to deducing that math is itself a liberal discipline, hence a natural component of liberal education.

To see why mathematics is a liberal discipline, one must know what mathematics is, and what a liberal art or science is. We will not need an exact definition of mathematics in order to reach our conclusion. Everyone's rough-and-ready notion of what mathematics is should suffice. It is enough if we recognize geometry and number theory as examples of pure mathematics, and if we recognize other mathematical disciplines as well that apply mathematics in some way, such as astronomy, modern physics, and the mathematical analysis of music.

We could spend a lot of time talking about what an art is and what a science is, but a general understanding of these, too, is sufficiently accurate for my purposes. An art, let us say, is a reasoned knowledge of how to achieve or make some particular human good, and a science is an ordered body of sure conclusions about some special subject matter.

I cannot so easily get away with calling some arts and sciences liberal without an explanation. Our word *liberal* is from the Latin *liberalis*, itself from *liber*, meaning "free." And what does it mean to be free? Those who are free can be identified by contrast with those who are not free, namely, prisoners and slaves. Prisoners and slaves live a certain way not because it is somehow naturally fulfilling for them to live that way, but because someone else forces them to live that way in order to serve or preserve a desirable way of life for others; they themselves are not permitted to share in the desirable way of life that they are made to serve. Now an education might help someone to get out of prison on good behavior, or to reintegrate into society after being let out. Or an education could help someone escape or avoid the slavery of a sweatshop. But an education is called “liberal” not because it frees us from such institutional forms of slavery or imprisonment, but because it helps free us from the sort of slavery and imprisonment that are natural to all mankind.

Here is what I mean. Like slaves or prisoners, who are in a special condition preventing them from living as they are naturally inclined to live, we all find ourselves born in a common condition preventing us from living as we are naturally inclined to live. All of us by nature desire to be good and happy human beings; yet we are born with a certain tendency to vice and to the misery it brings, and born also in ignorance both of what our goodness and happiness consist in and also of how to achieve them. And there is more. All of us by nature desire to know, to grasp the whole world around us within our souls, to be able to contemplate it whenever we wish, rather than to be stuck with dark and empty minds like prison cells without windows; and yet we are born in total ignorance of the world, and when we try to understand it, we find ourselves more prone to error and mental paralysis than to easy and error-free discovery of truth. In short, we desire to be free from ignorance, error, vice, and misery, but find that these evils come far more easily to us than their opposites.

An education in which we learn truths we naturally desire to understand liberates us from ignorance and error concerning such truths, and for that reason is called a liberal education. Learning truths we must understand in order to become happy and good also deserves to be called a liberal education, though for a somewhat different reason. A knowledge of the right way to live sets us free from a paralyzing ignorance, in which we cannot wisely direct ourselves in our own actions, leaving us condemned to a trial-and-error approach to living human life—a way of life involving lots of trials, and lots of painful errors.

A complete liberal education therefore has two principal parts. The first is called theoretical or speculative philosophy. This part frees us from ignorance and error concerning truths we naturally desire to know for their own sake. It is called "theoretical" or "speculative" not in the sense that it is uncertain and susceptible to being wrong—that would hardly liberate us from ignorance and error—but in the sense that it does not direct human action, but simply looks at truths worth seeing. Hence the words *theoretical* and *speculative* come respectively from the Greek and Latin words for "looking." This part of education is called "liberal" because it frees us from ignorance and error regarding things our minds naturally desire to look at and to see. Such an education is sufficient by itself to liberate our minds in that sense.

The second principal part of liberal education is called practical philosophy, or else moral or political philosophy. This part frees us from ignorance of the way to live in order to become good and happy human beings. It is called “liberal” because it frees us from the inability to direct ourselves toward, and within, a life worth living, by freeing us from the ignorance and error that can trap us in vice and misery. Such an education is sufficient to free us from that ignorance. It is also called liberal because, in freeing us from certain things that lead to vice and misery, and in equipping us with knowledge that can help us direct ourselves to virtue and happiness, it can be said to some extent to free us from vice and misery themselves—this, however, it cannot do all by itself. We also need grace, and many other things, in order to rise up out of vice and misery. Those things, however, are goods outside of education. Of all the types of education that exist, moral and political education alone deserves to be called liberal on the grounds that the knowledge it imparts of itself helps free us from vice and the misery that accompanies it.

Here I might mention sacred theology. Does that fall in the theoretical part or in the practical part of liberal education? Actually, it contains both theoretical and practical truth. As you will learn in your junior theology class here, theology is the only science that is formally[1] both theoretical and practical. Consequently, it is

[1] Logic, by contrast, though it too is both speculative and practical, is formally practical (i.e., it is defined as a knowledge of how to do or make certain things), and is speculative only in its end or purpose. That is, logic is by its nature a knowledge of how to produce certain works of human reason (e.g., definitions, arguments, etc.), and so it is formally practical. Moreover, its matter is practical, not theoretical; it is about

"liberating" both in the sense that it frees us from ignorance of things we naturally desire to know, and in the sense that it frees us from the inability to direct our action to what is good and what will make us truly blessed.

Note that the two principal parts of a liberal education are called "free," or "liberal," in distinct senses. The theoretical part of liberal education is called "free" because, like the life of someone who is free and is not a slave or prisoner, it is worthwhile in itself, not just for some other intellectual life or some other human good that it serves or makes possible. The moral part of liberal education, however, is called "free" because, like someone who is free and is not a slave or prisoner, it is self-directive. All other practical sciences, in order to be rightly used, must be used in accord with rules determined by moral and political science, since it does not pertain to the military or medical arts and sciences (for example) to say in what their right use consists. By contrast, moral and political philosophy (and moral theology) are not subordinated to any higher science telling them what they ought to do or what their right use consists in, since there is no science that makes a more universal consideration of the good for man than these sciences, or which

things to be made or done, for the sake of making or doing them well, and is not about things in themselves worthy of being known. But the end that its works are meant to serve is precisely knowledge, not some further purpose to which knowledge might be put (even when the knowledge which logic assists us to acquire is put to a further purpose, such as building a house, logic does not assist us with that further action), and in that sense it is speculative in its end.

considers a greater and more ultimate good for man. Hence those who govern a city or nation are not meant to be experts in carpentry, or medicine, or even in the military arts, but instead are supposed to be wise in how to use these arts for the good of the whole city or nation, toward its justice and happiness. Accordingly, political philosophy and prudence are called master-arts, or "architectonic," in relation to all the other arts and practical sciences.[2]

Now if that were the whole story, liberal education would simply be an education in theoretical and political philosophy and in sacred theology. And that is very close to being true, since those are the principal parts of liberal education. But those disciplines are so astonishingly difficult for the human mind to learn that it cannot leap straight into them. Certain preparatory disciplines must be learned first, as a way into liberal education. These preparatory disciplines are readily accessible to those beginning a liberal education. One reason for their accessibility is that they are all arts. Arts deal with what we do or make by some plan we have in mind, and such things we know best. Besides being accessible to us, the preparatory arts I am describing also provide certain inroads into the principal and more difficult parts of liberal education, by

[2] Cf. the commentary of St. Thomas Aquinas on Aristotle's *Nicomachean Ethics*, Book 6, Reading 7, paragraphs 1195–1198 in the Marietti edition, and his commentary on Aristotle's *Metaphysics*, Book 1, Reading 1, paragraphs 24–28 in the Marietti edition.

equipping us with the general tools and methods we need in those sciences. Logic, for example, teaches us the right way to make definitions, statements, arguments, and the like, which are the very stuff of philosophy.

These preparatory arts are also called "liberal," but for two other reasons, different from, though related to, the reasons why the parts of philosophy are called liberal. First, these preparatory arts are called "liberal" because they exist for the sake of serving knowledge that is "liberal" in one of the ways I explained earlier. Now that might sound odd. How can something be called "liberal," that is, free, by reason of the fact that it serves something else? That makes it sound servile, not free. But consider: the king is "royal" in the first sense of *royal*; he is his royal highness. But his carriage is called the royal carriage, and his guard the royal guard. These are "royal" not in the sense that they are kingly in themselves, but in the sense that they are dedicated specifically to the purpose of serving the king. So too the liberal arts in general are not all "liberal" in the chief sense of being some part of theoretical or practical philosophy, but they are all "liberal" at least in the sense that they are dedicated specifically to the purpose of serving and leading to such liberal knowledge.

Moreover, the manner in which they serve those higher disciplines presents another reason why they deserve to be called "liberal." The products of these preparatory arts are made immediately by the soul, within the soul, and for the soul, as opposed to an art such as carpentry, whose products are formed immediately by the hands and tools of the carpenter, and in a physical material (i.e., wood), and in answer to a need of the human body (e.g., shelter). In contrast to that, consider the art of logic. Although it makes arguments out of words, which are vocal sounds outside the soul, it assembles these only for the sake of a further product, namely, a corresponding assembly of thoughts in the mind. If an assembly of spoken words could not produce a corresponding assembly of thoughts within the mind, the logician would have no interest in words. The logician's work thus serves a need of the mind or soul, not a need of the body. Now the soul is the part of man by which he is free;[3] by having a mortal body, man is in many ways enslaved, burdened with needs, subject to sickness and death. Socrates compared the human body to a prison in which the soul is trapped.[4] That is not the whole truth about the relationship between body and soul, but it is an important part of the truth, and it gets truer and more important the older one gets. An art that produces its works by means of manual labor, in corporeal materials, all in order to serve a need of the body, may

[3] "The body is subject to the soul in a servile manner, and man is free according to his soul," St. Thomas Aquinas, *Summa theologiae*, 1–2, q.57, a.3, ad 3.

[4] *Phaedo*, 81e. Cf. 83d as well. Also, from 66b to 67b he describes the soul as being enslaved by the body, and at 67d he speaks of the soul as being shackled to the body.

be a very necessary art indeed, but it should be called a manual art, or an art in the service of necessity, not a liberal art. An art such as logic, whose products are made immediately[5] by the soul, within the soul itself, and for the good of the soul, deserves to be called "liberal," since its work is made by, is made within, and is made for the part of man by which he is free.[6]

Certain arts, therefore, are called "liberal" at least for these two reasons: One, they are preparatory for higher liberal studies, and Two, their works are within the soul. Logic in particular is perhaps called "liberal" for an additional reason; like the one who is free, it is to some extent self-directive. It gives directions to other sciences regarding the general rules about making distinctions, definitions, arguments, and the like, and when making these things itself, it does not follow orders from others, but simply applies its own rules.[7]

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[5] The thoughts of the human soul are formed by the intellect, which is a power of the human soul alone, not of the soul and body together. They are not the products of cognitive powers in the body, such as sense and imagination, except in the sense that the objects of sense and imagination supply the intellect with the material for its object. Cf. *Summa theologiae*, 1, q.84, a.6, co., end. Even those liberal arts that produce works within and partly by means of the imagination, as geometry does in one way and literature does in another, can be called liberal in this sense, however, since imagining is a work of the soul more than of the body. Imagining can hardly be called labor in an ordinary sense; its product remains within imagination itself, and it is relatively effortless.

[6] Here one might mention fine arts, which are in certain ways between the arts of necessity and the liberal arts. Fine arts in many cases produce works by means of the body, though not to serve the necessities of the body, but in order to delight the soul. Painting and sculpture, for example, form products in corporeal materials, but not in order to address the needs of the human body. Fine arts at one extreme come close to the arts of necessity or even overlap with them, as we see in architecture and the art of building. At their other extreme, they come close to the liberal arts or even overlap with them, as we see in the literary art.

[7] Is the logician therefore in a position to give orders to the philosopher? Is the logician wiser than the philosopher? No, since the philosopher supplies the logician with the first and most general principles of logic by explaining, ordering, and defending them, whereas the logician does not do this for the philosopher. The logician assumes, for example, that we need definitions and arguments. But why is that

With all these things before our minds, we are now ready to see that mathematics is a natural part of liberal education. The reason is that mathematics is both a theoretical science and a liberal art.

That mathematics is a science is clear enough. As anyone knows who has studied a little mathematics, it is an orderly, sure, and reasoned understanding of necessarily true conclusions about a certain subject matter. The purely mathematical disciplines, such as geometry and arithmetic, are a perfectly sure knowledge, since they deduce their conclusions from necessarily and self-evidently true first premises. The applied mathematical disciplines, such as modern astronomy and physics, though less certain than pure mathematics, attain a certainty of a sort, and they are a knowledge of conclusions[8] in the sense that they confirm hypotheses about nature by putting them to the test of experience. So these mathematical disciplines are all sciences.

What is less evident is that mathematical sciences are theoretical in nature. What is most known about mathematics to those little acquainted with it is that it is extremely useful. Though that is true, the practical use of mathematics is only an application of mathematics to something else outside it, not what mathematics

so? The philosopher explains that. The logician assumes, too, that we have self-evident principles available to us. But where do these come from? It is the philosopher, not the logician, who answers that question. The philosopher follows the rules determined by the logician, but those very rules are determined in the light of principles which the logician merely accepts and uses, but the philosopher understands.

[8] Cf. St. Thomas Aquinas, *Summa theologiae*, 1, q.32, a.1, ad 2.

itself is about. The Pythagorean theorem is useful for squaring up a deck I am building; but it is not inherently about squaring up a deck. In fact, the demonstration of the Pythagorean theorem is quite useless for that purpose; it is only the conclusion that matters, and even then the universality of the conclusion is unimportant. It is good enough for a carpenter if a 3-4-5 triangle must be right; he need not know that every triangle whose longest side is equal in square to the sum of squares on its other sides must be right. If mathematics were essentially about its practical applications, it would not be much interested in proofs, or even in the universality of its statements.

Mathematics is inherently theoretical. Although it is in some sense about an order produced by reason, such as the order of steps in constructing a figure, or in carrying out an algorithm to find a number of some given description, it is chiefly about relationships that human reason does not make but only discovers and understands.[9] The relationship of equality between the square on the hypotenuse and the sum of the squares on the remaining sides of a right triangle, for example, is not a relationship the human mind establishes, but only finds. Geometry, therefore, and mathematics

[9] Hence mathematics is related to mathematical order in the first way distinguished by St. Thomas Aquinas in his commentary on Aristotle's *Nicomachean Ethics* (Book 1, Reading 1), i.e., mathematical order is something that mathematics contemplates but does not produce. Although mathematicians (and students!) in a sense can produce an individual right triangle, or construct individual instances of the perfect solids, or an individual series of prime numbers, no mathematician causes a right triangle to have its Pythagorean property, or causes the order of edge-lengths of perfect solids inscribed in the same sphere, or causes 5 to be the next prime number after 3.

generally, is a theoretical, not a practical, science. The order it aims to reveal is not produced by the human mind, but is intelligible, and knowing that order perfects the human mind, and so that order must in fact be the reflection of a mind superior to the human mind,[10] and for that reason mathematics is theoretical, and can even to some extent be called a certain philosophy.[11] Any science that studies certain universal principles of the good and the beautiful, as indeed mathematics does,[12] is a sort of wisdom. Mathematics, therefore, is a theoretical science, even a theoretical philosophy, and is in that sense "liberal."

Note that mathematical science is "liberal" in the sense that it is theoretical in its subject-matter, not merely, like logic, in the sense that its products are incorporated into some other theoretical

[10] The reason is that nothing is perfected by what is inferior to itself, except insofar as the inferior is a participation in something superior. St. Thomas Aquinas explains: "The entire consideration of the theoretical sciences cannot reach further than the knowledge of sensible things can lead. Now the knowledge of sensible things cannot constitute the beatitude of man, which is his ultimate perfection. For nothing is perfected by what is inferior to it, except insofar as there is in the inferior some participation of a superior. And it is clear that the form of a stone, or of any sensible thing, is inferior to man. Hence the intellect is not perfected by the form of a stone insofar as it is such a form, but insofar as in it there is participated some likeness of that which is above the human intellect, namely an intelligible light, or something of the sort. Now everything which is through another traces back to what is through itself. Hence it is necessary that the ultimate perfection of man be through a knowledge of a thing which is above the human intellect. Now it was shown that it is not possible to arrive at a knowledge of the separated substances, which are above the human intellect, through sensible things. Hence it remains that the ultimate beatitude of man cannot be in the consideration of the theoretical sciences. Rather, just as in sensible forms there is participated a certain likeness of superior substances, so too the consideration of the theoretical sciences is a certain participation of true and perfect beatitude," *Summa theologiae*, 1–2, q.3, a.6, co.

[11] Aristotle says there are three theoretical philosophies: mathematics, natural philosophy, and theology (*Metaphysics*, Book 6, Ch.1, 1026a19).

[12] See Aristotle's remarks concerning mathematics and the good and the beautiful at *Metaphysics*, Book 13, Ch.3, 1078a31–1078b6. St. Thomas Aquinas also remarks that "it is false to say that there is no good in mathematical things, as he [Aristotle] himself proves later in Book 9" (*Sent. Meta*., lib.3, lect.4). St. Thomas is referring to *Metaphysics*, Book 9, Ch.9, 1051a21–end. See also *Sent. Meta*., lib.9, lect.10.

knowledge. Hence mathematics is more liberal than logic, since it is a liberal science, whereas logic is not a liberal science (that is, a theoretical science), but only a liberal art.[13] Logic is also called a "speculative art," though not because it is about truths in themselves worth knowing, but because its end is to serve knowledge itself (and not something else beyond knowledge).[14]

Very well, then, mathematics is a theoretical science, and so it is part of a complete liberal education. Is mathematics also a liberal art? Certainly. It teaches us how to make certain things; how to make equilateral triangles, for instance, and squares, and perfect solids. It also teaches us how to find certain things, such as the center of a circle or the least numbers in given ratios. So it is an art.[15]

Moreover, it is liberal, since its products are not made of sensible

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[13] Logic is more liberal than mathematics in another sense of *liberal*, however. Insofar as what is self-directive and also directive of others is like a free man, logic is more liberal than mathematics, since logic is directive of mathematics, not vice versa. For that reason, it is called *ars artium* (St. Thomas Aquinas, *Exp. Post. An.*, lib.1, lect.1). It is also more like first philosophy than mathematics is in certain important respects: like first philosophy, logic is about things we cannot sense or imagine, and is about the most universal things. Simply speaking, however, mathematics is the more liberal discipline; first philosophy is directive of other sciences in some ways, but it is called the most liberal of all sciences not for that reason, but chiefly because it is about truth worth knowing for its own sake and not for anything else. Mathematics is about truth worth knowing for its own sake (even if also for the sake of other things), too, and it forms its products for the sake of its own scientific understanding. Logic forms its products for the sake of mathematics and other such sciences; mathematics in no sense forms its products for the sake of logic. Teachers of logic sometimes employ mathematical examples, but it does not follow from this that mathematics is for the sake of logic or at its service, any more than it follows, from the fact that the logicians sometimes uses examples drawn from natural science, that natural science is subordinate to logic.

[14] Logic, in other words, is "speculative" in that it serves a speculative end (i.e., knowledge itself, vs. any further thing that might be gained by means of knowledge), not by having an essentially speculative subject matter.

[15] Mathematics and logic are not "arts," however, in the same sense in which carpentry is an art. Although they are called arts in a less perfect sense than carpentry is, they are more perfective of man (*Summa theologiae*, 1–2, q.57, a.3, ad 3).

materials, but remain within the mind of the mathematician, and they are made not to serve the needs of the body, but to be subjects of theoretical understanding. Things assembled by the logician, such as definitions and arguments, are only *instruments* of theoretical knowledge in *other sciences*. Things constructed by the mathematician, however, are *subjects* of theoretical knowledge *in mathematics itself*. Mathematics is therefore more liberal than logic, even as an art. Geometry constructs the five perfect solids, for example, and these are the subject of geometry's own theoretical knowledge of them.

Probably it is only the most elementary parts of mathematics that should be called "liberal arts," since that phrase implies something easily learned by beginners, which is not itself a higher study, but only preparatory for higher studies. Einstein's theory of relativity is mathematical, is art, and is science, but it is not right to think of it as an easy introduction to the life of the mind for beginners. It is truer to think of it as a mathematical part of the philosophy of nature.

It should now be clear that mathematics is a natural part of a complete liberal education. Here is the reasoning:

Mathematics is both a liberal science and a liberal art;
Any liberal science or liberal art is part of a complete liberal education;
Therefore, mathematics is a part of a complete liberal education.

In this way, one sees that mathematics rightfully holds some place in liberal education. But that is not enough to show that mathematics deserves to be studied throughout an undergraduate formation in liberal studies, or to show why some exposure to modern mathematics (e.g., up to the nineteenth and twentieth centuries) is desirable, even necessary, for certain programs in liberal education. Those things should become clear once we see that elementary mathematical disciplines serve all the higher liberal sciences in various ways. That is the second part of my thesis, and so we move on, now, to the second, inductive, part of the talk.

Part 2: Mathematics as Ordered to Higher Liberal Learning

"Liberal education" is not identical with "education in the liberal arts," for two reasons. First, because the expression "liberal arts" refers only to introductory disciplines or introductory parts of them.[16] Hence there are many arts that are liberal, yet are not "liberal arts" in that restrictive sense. Abstract algebra (Galois theory), for example, might be termed an art, and it is liberal in the sense that it is a knowledge theoretical in nature, but it cannot be called a "liberal art" in the sense of being an introductory discipline that can and must be learned before going on to the higher parts of philosophy. It is far too advanced to be considered introductory or elementary for an undergraduate education in liberal studies as a whole, and it is not needed in order to understand, say, metaphysics or theology.

A second reason why liberal education is not identical with learning the liberal arts is that the most liberal of all knowledge is not an art at all, since it is in no way a knowledge of something made by reason. The knowledge of the Trinity imparted in theology, for example, is not a knowledge of anything made by reason, but of something by which human reason was made, and so Trinitarian

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[16] "The seven liberal arts do not sufficiently divide theoretical philosophy. However, some number the seven together while omitting the others because, as Hugh of Saint-Victor says in 3 of his *Didascalion*, those who wish to learn philosophy are educated in them first, and so they are distinguished into the trivium and the quadrivium 'because by these, as by certain ways, the lively soul may enter into the secrets of philosophy.' This also harmonizes with the words of the Philosopher in *Metaphysics* 2, who says that the method of science ought to be investigated before the sciences; and the Commentator in the same place says that someone should learn logic, which teaches the method of all the sciences, before the other sciences. These things pertain to the trivium. [The Philosopher] also says in *Ethics* 6 that mathematics can be learned scientifically by children, but not physics, which requires experience. And so one is given to understand that after logic math should be learned next, to which pertains the quadrivium. And in this way, by these, as by certain paths, the soul is prepared for the other philosophical disciplines," *Super De Trinitate*, pars 3, q.5, a.1, ad 3. This passage makes plain that St. Thomas and Hugh of Saint-Victor take the expression "liberal arts" to refer to a definite set of seven introductory arts. If St. Thomas and Aristotle thought that mathematics was entirely of an introductory nature, surely they would think differently today, after being made acquainted with higher mathematics.

theology is not an art. So too what Aristotle calls first philosophy, which is a philosophical knowledge of God, and of the universe as coming forth from God and existing for God, is not an art, since it is not about things made by reason. Nor is natural philosophy about things made by reason; it is about things made by nature. These most liberal disciplines are not arts at all, then, but only sciences. Hence they cannot be called liberal arts. Nonetheless, liberal education consists in the teaching and learning of them most of all, since they are the most liberal disciplines. An undergraduate liberal education, however, spends much of its time in the lower liberal disciplines, called "the liberal arts," because these preparatory arts are much easier to learn for beginners, and they provide a great deal of assistance in the learning and teaching of the higher liberal disciplines.

The various liberal disciplines, then, are not like so many animals in a zoo, things of the same general kind all living in the same institution but not having much more than that to do with each other. Instead, the liberal arts and sciences form a wonderfully unified and orderly whole. Not one whole science, or one whole art, but one whole education, one complete intellectual life for man. And there is a natural order in which they should be learned, the earlier ones being in various ways necessary for the learning of the later ones. In particular, the four "quadrivial" arts, namely geometry, arithmetic, astronomy, and music, must be learned prior to the higher sciences of natural philosophy, first philosophy, and sacred theology and ethics.

These mathematical liberal arts pave the way for the mind to enter into those higher sciences in two ways. In one way, they provide intellectual prerequisites, things that must be understood before one is ready to go on to the higher parts of philosophy. In another way, they foster and strengthen dispositions of will and emotion necessary for making a good beginning in the principal liberal disciplines. The remainder of my talk will be devoted to explaining these two ways in which the quadrivial arts contribute to the learning of philosophy and theology.

Part 2A: The Quadrivium as Intellectual Preparation for Higher Liberal Learning

The noblest, wisest, and most liberal of all sciences is sacred theology, which proceeds in light of divine revelation rather than in the light of reason. Because the divine light is so bright, however, for us it is a bit like staring at the sun, or at objects too brightly lit by the sun for us to see very well; we need the assistance of a dimmer light, more suited to our mind’s capacity, in order for divine truths to become as accessible to us as possible. The learning and teaching of theology thus relies on assistance from lower sciences that proceed in the light of reason. For that purpose, theology mainly employs the highest of the human sciences, since those bear most on the subjects of theology. Theology makes relatively little direct use of the quadrivial arts, but one can hardly find a page of St. Thomas's *Summa theologiae* without finding there some use of ethics, or political philosophy, or the philosophy of nature, or metaphysics.

We must therefore learn a good deal of philosophy before going on to theology. But even though these philosophical sciences proceed in the light of reason, they are still quite difficult for us, too. They are at the summit of what the light of reason can achieve. The philosophy of nature, for example, is difficult in part because it is about things almost too dim to see, things whose being and intelligibility is tenuous or impoverished, such as motion, change, time, matter, and the like. Reason must shine rather brightly to illuminate these things. First philosophy is even more difficult for us to learn, because it is about things too bright for us to see very well, such as the immaterial intelligences and God, and the highest universals such as being, unity, truth, goodness, and so on. None of these things is sensible or imaginable, and so they are not very close to our human mode of knowing. Practical philosophy presents its own challenges, since even our bad desires can get in the way of understanding its principles and conclusions, and since it concerns human affairs, which are extremely variable, and habits, which take a long time to develop and to manifest their consequences; the profitable learning of ethics therefore requires a long experience of life which young undergraduates must lack to one degree or another. Moreover, the kind of certainty attainable about moral matters is quite different from that to be expected in other sciences, and the great differences between one human being and the next (as opposed to one monkey and the next) can make it seem as though there is no objective order in the moral realm.

So we seem to be in a quandary. Prior to learning theology, we must learn the philosophical disciplines, but it does not seem possible for the human mind simply to begin with them, either, since they are so difficult. What to do? The solution is to begin by learning sciences that in some sense cover the same subject matters as these higher parts of philosophy, but that do so in a manner more accessible to our minds. In first philosophy one studies God and the order of the whole universe toward God in light of his causing all existence, and in light of how one good serves a higher good. In natural philosophy, one studies the universal cause of all change, and the inclinations of all natures toward what is good for themselves and for the universe. The human mind cannot begin with these ways of considering all things. But there are related and similar studies with which the human mind can begin. In its own way, the liberal art of astronomy (and in general the elementary parts of the mathematical science of nature that largely grew out of ancient astronomy), also studies the order of the whole universe, but in a manner more accessible to beginners. The order it studies is a mathematical order, a quantitative one; the priority of one natural quantity or of one quantitative natural law to another, for example, and the order of natural things in time and space. Though this order is less profound, it is still very rich and beautiful, and it is much easier to see and learn than any order in the nobility or purpose of things, or in the causes of their being.

In practical philosophy one learns how human reason can order human passions and actions toward the greatest good of man. Because of the difficulty of understanding that order, and the nature of the human good which is the cause of that order, it is best to begin with another study of order in the human passions. One finds this in the liberal art of music, which is a purely mathematical approach to the study of music,[17] and is only the more elementary

[17] Is even the liberal art of music also a fine art? St. Thomas Aquinas speaks of the liberal art of music as constructing melodies, after all. And it could hardly be called an art if it did not construct something—and what else would it construct but rhythms and melodies? And yet to do so seems to be the work of a fine art. This sort of confusion is soon cleared away, however, when one considers the liberal art of grammar. It teaches us how to "construct sentences" in the sense that it gives us the grammatical rules we must respect when we speak or write. But is grammar the same as the fine art of writing? Surely not. To make beautiful sentences, powerful sentences, or to tell a great story or compose a poem, is not the work of

part of it at that. Since music can embody, reflect, imitate, and also provoke various movements of the human soul, the liberal art of music is indirectly a study of intelligible order in human emotions, albeit a study less profound than the explicit study of their rational ordering to human happiness in moral philosophy (and also less profound than the study of music itself in moral and political philosophy).

What are some examples, now, of the ways in which astronomy and music, and the quadrivial arts in general, prepare the mind for the highest liberal disciplines? I will give five examples of that. The first example of how the quadrivium prepares the mind is this:

1. The Liberal Arts of Astronomy and Music Provide Evidence of Verifiable and Objective Order in the Universe and in the Soul

In the elementary mathematical study of nature and music we learn, before going on to higher studies of nature and of the human soul, that the universe really is intelligible and ordered as a whole. In going through the major movements of the history of astronomy and physics, for example, one sees that an outmoded theory of the

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grammar, but of another, higher art (or inborn ability). So too it is the work of the liberal art of music to show us the mathematical rules that good rhythms and melodies must respect, but the work of another art (or inborn ability) to produce beautiful music.

order of the universe does not get replaced by an admission of sheer disorder, but instead by a theory representing a more profound, more beautiful, and more all-embracing order than the one it replaced. And if order is thus discovered in the relatively superficial consideration of nature in light of its quantitative aspects, surely the universe will exhibit order when looked at in light of deeper and more substantial things, such as the order of its parts in their various degrees of being, life, intelligence, and goodness. Similarly, in music, if we find numerical order in tones, intervals, chords, scales, and melodies, which things merely evoke and reflect movements of the human soul, surely there must be an intelligible, rational order in the movements of the human soul themselves.[18] In these ways, the liberal arts of astronomy and music can establish a reasoned expectation in the mind of the student that the universe and the human soul must be intelligible also in light of non-mathematical considerations; a conviction, in other words, that there is such a thing as natural philosophy, first philosophy, and moral philosophy.

2. Elementary Astronomy and Physics Illustrate How the Order of the Universe Is not Made by Human Reason but

[18] Cf. Aristotle's observation of how strange it would be if artistic representations of lower animals were delightful, and yet there were nothing worth considering in the animals themselves (*Parts of Animals*, Book 1, Ch.5, 645a10–645a13).

Is Only Discovered by It

My second example of how the quadrivium intellectually prepares the mind for higher studies is this: elementary astronomy and physics convince the attentive student that the quantitative order in the universe is not made by human reason, but only discovered by it. These disciplines, moreover, strongly suggest that the mathematical intelligibility of things is due to some intelligence. Hungarian-American physicist and Nobel Prize winner Eugene Wigner wrote a famous article called "The Unreasonable Effectiveness of Mathematics in the Natural Sciences."[19] Indeed, if one assumes the universe does not come forth from any intelligence, especially from an intelligence having the very mathematically-inclined human intellect in mind, it is difficult to give any reason why extremely abstract mathematics originally developed by itself, without looking to nature, has so frequently turned out to be somehow embodied in the world around us. Similarly, in the liberal art of music, one can see that the numerical properties of tones, intervals, scales and the like, are only discovered by reason, not established by it, and that these numerical properties are somehow rooted in nature itself, both in

[19] Originally the Richard Courant lecture in mathematical sciences delivered at New York University, May 11, 1959, the article was subsequently published in *Communications on Pure and Applied Mathematics*, 13:1–14, 1960.

the nature of sound and in human nature. This natural order must also be respected by reason if it is to construct effective melodies. These facts tend to engender in the minds of students a conviction, rooted in experience and rational insight, that reason's ordering of human passions themselves, not just of their musical imitations, must also respect a prior natural order. In other words, the mathematical study of music, done well, shows students an intelligible, natural, and objective aesthetic order, despite all the usual relativistic objections concerning beauty. This discovery of mathematical order in sounds that are beautiful and that move the human soul prepares students to discern in some analogous fashion natural and objective standards of moral order.

Now a third example:

3. Elementary Astronomy and Physics Correct Errors in the Older Science of Nature, Enabling Us to See More Clearly What in Aristotle's Philosophy of Nature Was Mere Theory, and What Is Timeless Truth

Much of the philosophy of nature has come down to us from Aristotle and his greatest commentators, such as St. Thomas Aquinas. These thinkers, however, predated modern science, and consequently labored under many misconceptions about the natural world; they were geocentrists, for example, and held that there were four elements, and that the species of living things did not appreciably change over time. Learning the basic lessons of modern science puts students in a better position to discern what in the philosophy of Aristotle is philosophical truth, and what in it is really a dated attempt to answer questions better addressed by a more mathematical or hypothesis-testing science of nature. (Conversely, learning the philosophy of Aristotle puts students in a better position to discern what in modern discourse is truly science, and what in it is really just poorly-done philosophy, far behind the wisdom of Aristotle.)

Now a fourth example of the intellectual preparation provided by the quadrivial arts:

4. Experience of the Quadrivial Arts Exposes the Limitations of the Quantitative Study of Nature and the Human Soul, Showing the Need for Another Knowledge of Them

Modern physics is so powerful and so all-pervasive it is not always easy to see how it is not in fact just the whole truth about nature; there seems to be no part of nature we can point to that physics cannot explain. But the liberal art of music gives us a clue about how to interpret that. There is no part of a melody that cannot be subjected to numerical analysis. And yet everyone who has learned this sort of analysis sees that it is very far from a complete understanding of a melody. Not only that, the mathematical understanding cannot possibly be the best and deepest understanding of music, since its language is purely numerical, and leaves out of its account the goodness or badness of music's effect on human passions. Similarly, the art of grammar helps us see that there can be an account of an entire thing without it being the entire account of that thing; all of *Hamlet*, for example, is intelligible and explicable in light of a grammatical analysis, but it does not follow that a grammatical understanding of *Hamlet* is a complete understanding of *Hamlet*. A relatively superficial study of music, then, is not only easier because it is relatively superficial, but also beneficial for being so clearly inadequate, since it points the way to a higher, deeper understanding both of music and of the human passions it embodies and imitates.

As for the remaining quadrivial arts, geometry and arithmetic, and other elementary parts of pure mathematics that were developed later, such as analytic geometry and basic calculus, these belong to liberal education as liberal arts in their own right, but also, and more so, for the sake of the higher quadrivial arts already mentioned. Geometry is ordered to astronomy and physics, in which it is applied in many ways, and arithmetic to music, in which tones, intervals, and scales are found to exhibit numerical properties. Geometry, then, is liberal in itself, but is also ordered to astronomy and physics, which are liberal in themselves but are also ordered to natural philosophy, which is liberal in itself but is also ordered to first philosophy or metaphysics. Arithmetic is liberal in itself, but is also ordered to music, which is a liberal art in itself, but is also ordered to moral philosophy, which is liberal in itself, but is also ordered to first philosophy. First philosophy, and all of theoretical philosophy, is in turn ordered to the theological study of God and creation, and moral philosophy is ordered to moral theology. There you have, in a nutshell, a large part of the order underlying our program of studies here.

A fifth example of how the quadrivial arts prepare the mind for higher studies is that

5. Mathematics Supplies Its Learners with Aids for Understanding Profound Truths in Philosophy and Theology

Geometry and arithmetic, for instance, supply an abundance of examples, contrasts, precedents, and analogs, from which theology teachers can draw when helping students to understand certain truths about God and the angels. Mathematics was used in this way even before Christianity. Plato, for example, employs his famous illustration of the divided line,[20] a geometrical construction, in order to teach us something about the higher beings, how they are known, and how they differ from the beings most familiar to us in sense experience. Moreover, Plato's Socrates is always seeking to grasp the forms of things and to express them in definitions, and our first scientific encounter with "forms" is with the shapes of things in geometry.[21] Tradition has it that Plato had engraved at the entrance to his Academy the words *No one ignorant of geometry may enter*,[22] as if to say that prior to studying the forms of things in philosophy, one must have studied the shapes of things in geometry.[23]

[20] Plato, *Republic*, Book 6, 509d–511e. Note that immediately after Plato gives us the divided line, he gives us the parable of the cave, which has much the same purpose. The first is a use of geometrical imagination, and the second a use of poetic imagination, in order to assist the reader into understanding divine things.

[21] Plato, like the Pythagoreans before him, also noted the likeness between numbers and the essences or substances of things. In the case of corporeal things, there is more than a likeness, since number enters into the definitions of things in some way, as we can see not only in the definition of body in general ("substance having three dimensions"), but even in the definitions of the elements, which are practically defined by their atomic numbers. Aristotle, too, noted the likeness between numbers and essences of things (see *Metaphysics*, Book 8, Ch.3, 1043b–1044a14).

[22] The words, according to some sources, were: Ἀγεωμέτρητος μηδείς εἰσίτω.

[23] Proclus, in his introduction to his commentary on the first book of Euclid's *Elements*, mentions that "Plato teaches us many wonderful doctrines about the gods by means of mathematical forms" (Proclus, *A Commentary on the First Book of Euclid's* Elements, translated, with introduction and notes, by Glenn R. Morrow, Princeton University Press, Princeton, NJ, 1970, p.19). Unfortunately, Proclus provides us with

One example of how elementary mathematics can assist the teaching and learning of divine truths is as follows. A higher angel understands truth by means of more universal concepts than a lower angel can hold in his mind.[24] In other words, a higher angel can know all the same things a lower one can know and more besides, and with fewer thoughts. Now that might sound impossible. How can someone know more things while using fewer thoughts? A teacher of theology can help us overcome that difficulty by pointing to something analogous in geometry: a geometric figure with a smaller perimeter can hold more area than another figure with a greater perimeter. That too might sound impossible. How can a figure with a smaller boundary contain a greater area? Well, compare a 1-by-30 rectangle and an 8-by-8 square, and you will see that the square has significantly less perimeter, yet holds significantly more area. The reason is the greater uniformity and simplicity in the way the square uses its perimeter. Now the words we speak and the thoughts they express are to the truth they contain as the perimeter of a figure is to the area it contains. So just as the lesser perimeter can contain a greater area, if it is simpler and more unified, so too fewer words or fewer thoughts can contain more truth, if only they are somehow

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no examples, but we should be able to provide our own, since Christians know more about divine things than Plato did.

[24] *Summa theologiae*, 1, q.55, a.3.

simpler and more unified. Hence the mind that naturally knows all things through fewer thoughts is superior to one that naturally knows all things through more thoughts. And just as the most perfect boundary, namely, a circular one, is also the simplest and most uniform, so too the most perfect mind, namely, the divine mind, is the simplest and most unified, containing all truth whatsoever in a single thought.

Part 2B: The Quadrivium as Dispositive Preparation for Higher Liberal Learning

So far we have looked at some examples of ways in which mathematics prepares the mind to ascend to higher liberal studies. As I mentioned earlier, however, the quadrivial arts also cultivate dispositions of will and emotion that are in various ways and degrees necessary for making a good beginning in, or making good progress in, the higher liberal disciplines. Let’s see that now, in examples.

First of all,

1. Mathematics Cultivates a Love of the Beautiful

One disposition of will and emotion prerequisite to higher liberal studies is the love of the beautiful. We all love the sort of beauty we can see with our eyes, but intelligible beauty is a subtler thing, and unless we come to see it and love it, we will not long persevere in the life of the mind. Mathematics cultivates this love, and conducts us from the beauty we see with our eyes to the type we can grasp in our imaginations and on to the sort we behold with our intellects.

Mathematics is actually driven by the love of the beautiful. When you study Euclid, you might ask yourself why Euclid does not stop in the middle of a proposition somewhere, when he has proved a step on the way to his final conclusion; why not make the middle step itself a final conclusion? Who made that choice, and based on what criteria? Euclid made that choice, based partly on pedagogical criteria, but also based on what is most beautiful. Mathematics aims to prove beautiful truths about beautiful things by means of beautiful proofs.

Accordingly there are three locations of beauty in mathematics. One is the mathematical things themselves, such as regular polygons inscribed in a circle, exhibiting their admirable exactness and symmetry, which are ingredients or forms of beauty. Another is the truths about such things, for example, the lovely truth that if a regular polygon is inscribed in a circle of unit radius, then the product of all the chords drawn from any one vertex to all the other vertices is equal to the number of sides in the polygon (a truth you should encounter in your senior year). The third location of beauty in mathematics is in its proofs. For example, of the dozens and dozens of different proofs of the Pythagorean theorem, some are laughably long, complicated, and difficult, while others are quite economical, delightfully transparent, employ a simple and pretty construction, and clearly bring out the reason for the truth of the conclusion.

Like the wise evaluation of poems, myths, epics, and plays, the study of mathematical things is a beautiful study of beautiful things. And the beauty of mathematical things is accessible to our imaginations, so that it is easier to appreciate than moral or spiritual beauty. And while literature helps us to see and love moral and spiritual beauty, which are deeper things than the beauties of mathematics, the beauty of mathematical argumentation and problem-solving is closer to the beauty of philosophical and theological demonstrations than is the beauty and order of a play.[25]

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[25] On the other hand, the beauty of the things a play, poem, or novel is about might well be more like the beauty of the things philosophy is about than the beauty of mathematical things could ever be. Also, there

2. Mathematics Develops Our Sense of Wonder

Mathematics strongly encourages another disposition necessary to the whole life of the mind: a sense of wonder. Wonder is the desire to know a truth that is in itself desirable to know. Without wonder, we would have interest only in truths about how to live, how to make a living, and how to acquire the necessities and pleasures of life. Truths that perfected the human mind, but did not equip us for doing, acquiring, and making things, would be completely neglected; there would be no philosophizing, no theoretical science.

In Plato's *Republic*,[26] Socrates remarks to Glaucon that the eye of the soul is "purified and kindled afresh" by mathematical studies "when it has been destroyed and blinded by our ordinary pursuits." It rouses and restores our wonder.

How does mathematics provoke our wonder? Unlike logic, it is about things worth knowing in themselves and accessible to our imagination. So we readily wonder about them.

For example, take a look if you would, at Figure 1 on the

is a remote similarity between the movement of a philosophical science proposing and resolving problems and the plot of a play involving its characters in complications and then resolving them in its denouement. Mathematics and literature train the soul for philosophy in complementary ways.

[26] Book 7, 527e.

handout. You are looking at a large circle in which six other circles have been drawn internally tangent to it at A, B, C, D, E, and F, and they are also externally tangent to each other. Apart from that, they can be of whatever size you please. If you now join AD, BE, CF, these straight lines must all meet at a single point, as you see in Figure 2. It always works. But why?

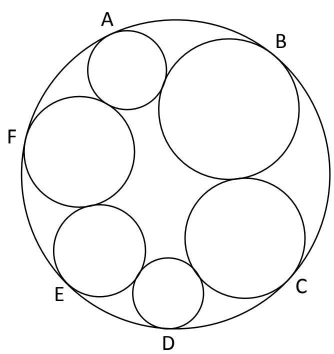

Figure 1

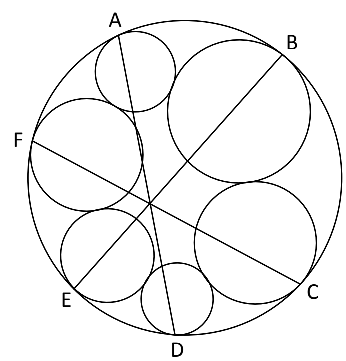

Figure 2

3. Mathematics Counteracts Skepticism and Intellectual Relativism

Another disposition of will and emotion that mathematics fosters is intellectual hope. We live in a post-modern age characterized, even defined, by a loss of confidence in reason's ability to answer any great questions about morality, about the human soul, about God. It is as if the would-be philosophers of our time, having studied so many conflicting philosophers and philosophies, and having seen that none has stood triumphant over the rest, have decided that philosophy is just words, or word-games and mind-games, not wisdom, and that arguments cannot get to the truth of things, but can only serve to manipulate others. That is the condition Socrates warns his friends about on the day of his death, after they begin to fall into despair of ever finding good arguments for the immortality of the soul.[27] Given the difficulty of knowing such things, we could easily become skeptics about them, simply by living and breathing in the academic and intellectual climate of our time. All too often, the result of higher education is moral and intellectual relativism, the belief that all beliefs about the greatest questions are created equal, since none is really knowable, or has any truth founded in a reality common to us all.

If, then, we are to learn any science higher than modern science,[28] if we are to take philosophy and theology seriously, we must find a way to counteract this powerful current of skepticism in our teaching and learning. We must find ways to encourage the hope of obtaining satisfying, reasonable answers to our great questions. Mathematics fosters such hope, by showing us we can obtain sure proof of non-trivial truths. And it fosters hope right from the start: Euclidean geometry is very certain, clean, exact, orderly, and non-trivial.[29]

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[27] *Phaedo*, 89b–91c.

[28] For various reasons, modern science is adept at getting its practitioners to agree with each other, and so people are less prone to skepticism and relativism in matters of modern science.

[29] I grant that it is quadrivial.

In particular, an education in philosophy cannot ignore modern philosophers. But one is more liable to fall into despair when reading them at length if mathematics, the clearest success of reason, has become a distant memory, or has only been superficially experienced.

4. Learning Mathematics Can and Should Promote Humility and a Healthy Fear of Making Mistakes

Of course, it is possible to go too far in one's confidence in reason, and especially in one's confidence in one's own reason. Mathematics helps with that, too. It imbues us with a healthy fear of mistakes. Mathematics helps develop a sense of just how provably, definitively, objectively wrong we can be, and how surprisingly often. It shows us that there are many different ways we can overlook something important, and how a tiny mistake can lead to a great number of enormous mistakes when it is carried into subsequent reasoning.

Mathematics teaches us to become aware of how prone we are to making mistakes, and does so without robbing us of our confidence that we can find the truth. How healthy that is, and how necessary for learning the higher disciplines. When pursuing them, we all too easily believe we have apprehended the truth when we have not, precisely because the truth in higher disciplines is so difficult that it is hard for us to see when we have missed it. We tend to think mathematics excels in catching us out in mistakes because it is the hardest of all subjects. That is quite wrong. Advanced mathematics is of course extremely hard indeed, and most of the mathematics in which today's mathematicians are engaged is too involved for most of us ever to understand. But elementary mathematics, relative to the rest of science and philosophy, is quite easy, is eminently learnable, which is why the Greek word for one who is fond of learning, μᾰθημᾰτικός, became the word for a mathematician in particular.

Mathematics can, and ought to, encourage a certain fear of error in us, and a certain humility, too. Both of these dispositions are even more necessary in the higher sciences. If we are capable of failing to notice our own errors when we are talking about relatively superficial things such as numbers and triangles, which things we can imagine, how much more susceptible to undetected error must we be in sciences that talk about invisible and unimaginable things.

I say that math "can" and "ought to" promote humility and a healthy fear of error, not that it does so infallibly. There are some in whom it might tend to have the opposite effect, namely, those of us who are (so to speak) too good at math for their own good. In the case of such souls, who rarely make mathematical mistakes and can conceal the ones they do make, there may be a danger that they will come to think of themselves as a species apart, in need of no assistance in learning the truth. But I think it is a real and objective danger only for the great world-geniuses of mathematics, God bless them. For anyone inferior to them—for example, any one of us in this room—a remedy for mathematical arrogance is readily available, namely, our manifest inferiority to the mathematical geniuses in the world, which should be all the clearer to us the better we understand mathematics. If you are quicker on the uptake than your fellow students in learning from Newton and Einstein, and you are tempted to be impressed with yourself, then look up for a moment, and see, towering above you, Newton and Einstein, and all the other geniuses from whom you are learning. You should notice your dependence on, and inferiority to, the intelligence of others. Which brings me to my next point:

5. Learning Mathematics Can and Should Foster a Willingness to Learn from Others

A natural consequence of humility is "docility" or teachability, a readiness to learn from others wiser than ourselves. In imitation of

Hesiod's division of the human race with respect to intelligence,[30] I can divide all of mathematics with respect to my own intelligence as follows:

- First, there is the mathematics that I can figure out by myself;
- Second, there is what I can't figure out by myself, but can understand when someone else explains it to me;
- Third, there is what I cannot understand even when someone explains it to me.

By far the largest of those three categories is the last. The next-largest category is the second, the one containing those parts of mathematics that I could never in a million years have figured out for myself, but which I could learn in a fairly short time from those intelligent enough to have discovered them for themselves. When I see myself so dependent on Euclid, Euler, Gauss, Cantor, Gödel, and countless others in order to come to understand so many beautiful things, I am apt to draw certain conclusions about myself. I am inclined to a truer assessment of my own ability and knowledge than I would have been without having seen how much insight I gain from their teaching, which insight I could never have gotten without them. Now, if I absolutely need such teachers in

[30] *Nicomachean Ethics*, Book 1, Ch.4, 1095b10–1095b13.

order to learn mathematics, in which I am learning about relatively superficial things such as numbers and triangles, which things I can imagine, how much more must I rely on great geniuses in order to learn sciences about things I cannot imagine?[31] Similarly, if I find I need subtle distinctions and long, complicated, difficult arguments involving many steps and stages in order to learn the deeper truths of mathematics, how much more will I need such things in order to see the deeper truths of philosophy?

6. Mathematics Introduces Our Minds to Counterintuitive Truths, Thus Training Us Against Too Hastily Resolving Questions in Light of What We Take to Be Obvious

Another necessary predisposition for learning the higher sciences is expecting the unexpected, an openness to counterintuitive truths. That disposition is important, since otherwise our minds will be closed off to the highest and deepest truths, many of which run contrary to what we initially expect or think to be possible. It is especially important in our program to provide some experience of

[31] Perhaps it is worth adding a qualifier: a genius in mathematical sciences need not be one in higher philosophical sciences, or vice versa. St. Thomas and Aristotle would surely have depended on the great minds of mathematics and physics, too, in order to learn the tremendous advances made in those subjects since their time. They would not therefore be disqualified from being geniuses of the first order in philosophy and theology. Nor were Newton, Gauss, Einstein and the like geniuses of any sort in first philosophy or theology. Still, even St. Thomas depended on Aristotle for many things, and Aristotle on Plato. Plato, incidentally, was a genius of the first order in philosophy, and was not too shabby a mathematician, either.

counterintuitive, seemingly impossible truths, since we lay so much stress on the importance of common experience and self-evident truths. We do this because common experience and self-evident principles are the soil out of which the most general parts of natural philosophy grow, and one must learn some of those parts of natural philosophy before one can learn any metaphysics, which also springs up from self-evident principles, even as mathematics does. Moreover, modern science, which dominates most people's understanding of intellectual life, is characterized more by its use of testable hypotheses than by its use of self-evident truths, and more by its use of highly specialized forms of experience than by its use of the types of experience common to us all. And modern philosophy often unjustly denigrates the use of common experience and of self-evident truths, or even denies that there are such things. For these reasons, then, we lay special stress on the importance and certainty of common experience and of self-evident truths.

But one can overestimate the power of common experience and of self-evident truths to settle our questions, or else overestimate our ability to recognize the true data of common experience and the truly self-evident things, and to tell these apart from the hasty assumptions we naturally make about things. To become liberally educated, we need to learn to strike a certain balance; to see, on the one hand, that common experience and self-evident truths alone suffice for answering some, but not all, important questions, and to see, on the other hand, that many things seem like matters of common experience, or like self-evident truths, but really they are nothing of the sort.

Here is an example of what I mean. In relativity theory, we learn the shocking, counterintuitive truth that velocities do not add up the way we normally think they do. Suppose you are standing in an airport terminal, watching me walk on one of those moving walkways. If the walkway is moving past you at 10 mph, and I am walking in that same direction at 3 mph relative to the sidewalk, how fast am I moving relative to you? Our intuitive answer is 13 mph. But that is wrong! Not very wrong, but still wrong. We won't worry tonight about why it is wrong, but it is wrong. So that is one extremely counterintuitive truth we come across in physics, where there is very little possibility of our dismissing the whole theory as nonsense, since it is so clearly demonstrated from facts of experience (not ordinary experience, however).

Counterintuitive truths of this sort learned in the mathematical disciplines set an important precedent for higher studies. If we can think that an incorrect way of adding velocities is the self-evidently correct way, then all the more will we be capable of thinking that a false or impossible way of understanding invisible or unimaginable things is a self-evidently true understanding of them. For example, it seems self-evident that if two things are each the same as some third thing, then they must be the same as each other. Now God the Father and God the Son are each the same as the one true God. Therefore, it seems, God the Father is the same as God the Son, and so these are not two different divine persons, but one and the same person under different names. That conclusion is of course contrary to Christian truth. But it also seems to follow quite readily from an apparently self-evident statement together with the Christian truth that there is only one God. We are better prepared to discover the subtle fallacy involved in this would-be refutation of the Trinity after having seen, in mathematical physics, subtle fallacies such as the calculation that says I should be moving past you at 13 mph.[32] The precedent in physics puts us on notice for similar things in theology.

Conclusion

Perhaps now both the meaning and truth of my answer to our original question have become plain. Why and to what extent should liberal education include mathematics?

[32] At Thomas Aquinas College, students learn the fallacy in the anti-Trinitarian reasoning before learning why the intuitive way of adding velocities is mistaken, but learn many other counterintuitive truths in mathematics and physics prior to studying Trinitarian theology with the help of St. Thomas Aquinas.

First, any liberal education program meant to offer a first look at the whole of liberal learning must include some mathematical disciplines, such as those making up the quadrivium, just because they are in fact liberal disciplines.

Second, an undergraduate program of liberal education aiming to provide a first look at the principal doctrines of theology and philosophy should include some exposure to the quadrivium, because of the many and diverse ways those lower disciplines, which contemplate the order in the universe and in the human soul in a quantitative and accessible manner, prepare students' minds and wills for a deeper and more difficult understanding of the universe and the human soul in natural philosophy, first philosophy, practical philosophy, and theology.

Third, a program of liberal education proposing to teach a fair amount of natural philosophy, as ours does (both for the intrinsic worth of that science and to pave the way to metaphysics and to a more fruitful study of certain parts of theology), must also include some amount of relatively modern mathematics and physics. This is profitable, and even necessary, for a couple of reasons. First, the great master of the philosophy of nature, of the first and most general parts of the science of nature, is Aristotle, and so we must turn to him to learn those parts of natural science; and yet Aristotle to some extent mixed into his general study of nature the theories of his day concerning the stars and the elements. As a corrective for these deficiencies in his understanding of nature, and in order to bring into sharper relief the timeless elements of his doctrine, one must look to more recent accomplishments of science, which in turn requires an upgrade in our mathematical toolbox. A second reason it is necessary to study some modern mathematics and physics alongside Aristotle's philosophy of nature, is to maintain a certain balance in one's understanding of how to study nature. With Aristotle, we see the great power of ordinary experience and self-evident truths in the study of nature, but not so much their limitations, whereas with modern science we get the complementary view of the great power of studying nature through testable hypotheses and extremely special forms of experience.

My brief sampling of these ways in which mathematical sciences assist the learning and teaching of superior sciences should not be taken to be exhaustive. But I hope that it was representative and persuasive. And I hope, too, that you enjoy and profit from the tiny slivers of mathematics that we study here, and that they bring joy and light to your mind.